AP Calculus BC Syllabus

Curricular Requirements

CR1	The students and teacher have access to a college-level calculus textbook, in print or electronic format.	*See page:* 4
CR2	The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.	*See page:* 2
CR3	The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes.	*See page:* 18
CR4	The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations.	*See pages:* 18-20
CR5	The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification.	*See page:* 20
CR6	The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation.	*See page:* 20
CR7	Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.	*See pages:* 5, 20
CR8	The course provides opportunities for students to use calculus to solve real world problems.	*See pages:* 20-21

Introduction

This course in the study of calculus is divided into two major areas: differential and integral calculus, together with the additional AP Calculus BC topics of parametric equations, polar coordinates, vector-valued functions, and infinite sequences and series.

The course is aligned with the **College Board AP Calculus BC Course and Exam Description** and is equivalent to **two semesters of college-level calculus** *(CR1)*. Students are required to take the AP Calculus BC Exam in May.

Students work with functions represented:

- graphically
- numerically
- analytically
- verbally

with emphasis on connecting these representations and communicating mathematical reasoning clearly *(CR4, CR6)*.

The course is organized around the Big Ideas:

Big Idea #1: Change
Big Idea #2: Limits
Big Idea #3: Analysis of Functions *(CR2)*

The College Board's CED is broken down into 10 units, and my course follows the sequencing/pacing of these 10 units. The three big ideas of calculus are included in the units as reflected in the CED. CR2

UNIT 1: Limits and Continuity (~3 weeks)

UNIT 2: Differentiation: Definition and Fundamental Properties (2–3 weeks)

UNIT 3: Differentiation: Composite, Implicit, and Inverse Functions (2–3 weeks)

UNIT 4: Contextual Applications of Differentiation (~2 weeks)

UNIT 5: Analytical Applications of Differentiation (2–3 weeks)

UNIT 6: Integration and Accumulation of Change (~4 weeks)

UNIT 7: Differential Equations (2–3 weeks)

UNIT 8: Applications of Integration (3–4 weeks)

UNIT 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions (~3 weeks)

UNIT 10: Infinite Sequences and Series (4–5 weeks)

Student Practice

Throughout each unit, Topic Questions will be provided to help students check their understanding. The Topic Questions are especially useful for confirming understanding of difficult or foundational topics before moving on to new content or skills that build upon prior topics. Topic Questions can be assigned before, during a lesson, or as homework. Students will get rationales for each Topic Question that will help them understand why an answer is correct or incorrect, and their results will reveal misunderstandings to help them target the content and skills needed for additional practice.

At the end of each unit or at key points within a unit, Personal Progress Checks will be provided as homework assignments in AP Classroom. Students will get a personal report with feedback on every topic, skill, and question that they can use to chart their progress, and their results will come with rationales that explain every question's answer. A class period is set aside to re-teach skills based on the results of the Personal Progress Checks.

An extra day each week is devoted to an appropriate calculator activity, multistep word problems, Topic Questions, Personal Progress Checks, and/or free response questions (FRQ's) from released AP Calculus BC Exams. Emphasis is placed on problem solving, using the calculus in new settings, and helping students to see the connections among the big ideas and the major themes in calculus. FRQs, which emphasize real-world applications of the calculus are selected for discussion on the Discussion Board on this extra day.

Course Delivery and Student Interaction

The course is also designed around the four Mathematical Practices in AP Calculus outlined in the 2019 CED including:

Practice #1: Implementing Mathematical Processes
Practice #2: Connecting Representations
Practice #3: Justification
Practice #4: Communication and Notation

This course is delivered online. Students access lectures, assignments, and resources through the course website.

Discussion-based work using the Discussion Board is a regular part of the course. Students:

- post complete solutions
- ask questions
- challenge reasoning
- explain concepts using correct notation

These interactions develop the four Mathematical Practices:

- justification
- communication and notation
- implementation of processes
- connecting representations

Course Objectives

At the end of the course, students should be able to solve a variety of real-world problems using limits, derivatives, integrals, and series. Students are shown the interrelationships of these four major themes/threads throughout the course. The course teaches the students how to communicate their mathematical reasoning using proper mathematical terminology in complete sentences. Students are instructed how to answer problems in the context of the problem, both verbally and in written sentences/paragraphs, using appropriate measurement units.

Prerequisites

All students who are taking AP Calculus BC have completed precalculus and have a firm understanding of:

- Functions – their graphs and behaviors
- Trigonometry
- Logs and Natural Logs
- Transformations and Translations
- The use of their graphing calculator to solve problems
- The value of the Rule of Four to solve problems (analytical/algebraic, numerical,
- graphical, verbal/communication)
- Transcendental Functions

These and other prerequisite topics/skills are briefly reviewed, as needed, during the year to help students make valuable connections between the big ideas.

Textbooks, References, and Materials (CR1)

Primary Textbook: Larson, Ron, and Battaglia, Paul. Enhanced WebAssign with *Calculus for AP.* 2nd ed. Boston: Cengage Learning, 2021.

AP Review Text: Howell, Mark, and Martha Montgomery. *Be Prepared for the AP Calculus Exam.* Andover, MA: Skylight Publishing, 2016.

Reference Books:

Hughes-Hallett, Deborah, Andrew M. Gleason, et al. *Calculus Single Variable*. 4th ed. New York: John Wiley & Sons, Inc., 2005.

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic.* 4th ed. Upper Saddle River, NJ: Prentice Hall, 2012.

Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. Emeryville, CA: Key Curriculum Press, 2005.

Lederman, David. *Multiple-Choice & Free Response Questions in Preparation for the AP Calculus (AB) Examination*. 10th ed. New York: D & S Marketing Systems, Inc., 2016.

McMullin, Lin. *Teaching AP Calculus*. 3rd ed. Brooklyn, NY: D & S Marketing Systems, 2015.

Best, George, and Sally Fischbeck. *AP Calculus with the TI 83 Graphing Calculator*. Andover, Mass.: Venture Publications, 1998.

Software:

Best, George. *Best Grapher*.

Bradford, William. *Calculus AB Test Bank*.

Desmos

Weeks, Audrey. *Calculus in Motion*.

- Previously Published AP Multiple-Choice and Free-Response Questions including the 1997, 1998, 2003, 2008, 2012 released AP Exams
- AP Professional Development Workshops and Institute materials
- AP Central® website and AP Calculus OTC
- TI-83+ and TI-84 graphing calculators

Students also use AP Classroom topic questions and Personal Progress Checks as part of regular practice and review.

Graphing Calculator and Technology (CR7)

Students are required to have individual access to an approved graphing calculator such as a **TI-83/84 (or equivalent)**. Students are required to purchase this graphing calculator as a requirement for class admission.

- Students are exposed to numerous calculus applets during the course.
- Students download a number of calculator programs from the Resources Page online, including programs for Riemann Sums, Area between two curves, Euler's Method, and Slope Fields. These programs are designed to help students visualize the various concepts and to get a deeper understanding of calculus.
- Students are instructed throughout the course of the Four Functionalities allowed on the AP Exam with the graphing calculator including:
  - Plot the graph of a function within an arbitrary viewing window.
  - Find the zeros of functions (solve equations numerically).
  - Numerically calculate the derivative of a function.
  - Numerically calculate the value of a definite integral.

- Explore or interpret calculus concepts.

- I instruct students on the various software packages to illustrate volumes of solids, slope fields, and accumulation.
- During the course, problems will be represented and solved in four distinct ways: analytically, numerically, graphically, and verbally. Students will use a graphing calculator to determine the value of various limits, to determine the value of a derivative at a point, to find the value of a definite integral, to graph a function in various windows, and to solve a variety of equations, as well as explore concepts such as the limit of a function at a point.

Example:
Students compare numerical and analytical derivatives to understand approximation and error.

Assessment

Students are assessed using several methods. I count the daily homework which is submitted as a scanned PDF file to their DropBox as 10% of a student's grade. The other 90% is a combination of quizzes, labs, projects, Discussion Board participation, and unit tests. I will use the Personal Progress Checks (PPCs) designed by the College Board as formative assessments during the course of the 10 units to help students and me better understand what concepts my students are struggling with. The unit tests contain a no calculator section and a calculator section consistent with the AP Calculus BC Exam. Weekly labs and discussions consist of graphical, numerical, and analytical components and a written conclusion as a journal entry. Free-response questions are graded similar to the AP Exam. A midyear exam is given at the end of the first semester. Just before the AP Exam in May, students are given an entire AP BC Calculus practice exam, which is graded like the actual exam using the scoring guidelines published by the College Board. This is counted as their final exam grade for the year.

Because the mathematical communication component is so important in this class, students are strongly encouraged to do test corrections for every exam and scan them into their DropBox. These test corrections are an integral component of the learning process for this AP course and will help students understand the required concepts, as well as how to effectively communicate their answers.

📊 Course Outline

Unit 1: Limits and Continuity

Topic	Skill
1.1 Introducing Calculus: Can Change Occur at an Instant?	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.2 Defining Limits and Using Limit Notation	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.3 Estimating Limit Values from Graphs	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.4 Estimating Limit Values from Tables	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
1.5 Determining Limits Using Algebraic Properties of Limits	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
1.6 Determining Limits Using Algebraic Manipulation	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
Complete Personal Progress Check MCQ Part A for Unit 1	
1.7 Selecting Procedures for Determining Limits	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).
1.8 Determining Limits Using the Squeeze Theorem	3.C. Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
1.9 Connecting Multiple Representations of Limits	2.C. Identify a re-expression of mathematical information presented in a given representation.
1.10 Exploring Types of Discontinuities	3.B. Identify an appropriate mathematical definition,

	theorem, or test to apply.
1.11 Defining Continuity at a Point	3.C. Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
Complete Personal Progress Check MCQ Part B for Unit 1	
1.12 Confirming Continuity over an Interval	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
1.13 Removing Discontinuities	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
1.14 Connecting Infinite Limits and Vertical Asymptotes	3.D. Apply an appropriate mathematical definition, theorem, or test.

Unit 1: Limits and Continuity

Topic	**Skill**
1.15 Connecting Limits at Infinity and Horizontal Asymptotes	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Check FRQ A for Unit 1	
1.16 Working with the Intermediate Value Theorem (IVT)	3.E. Provide reasons or rationales for solutions and conclusions.
Complete Personal Progress Checks MCQ C and	
FRQ B for Unit 1 Take Unit 1 Test	

Unit 2: Differentiation: Definition and Basic Derivative Rules

Topic	**Skill**
2.1 Defining Average and Instantaneous Rates of Change at a Point	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.

2.2 Defining the Derivative of a Function and Using Derivative Notation	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems. 4.C. Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
2.3 Estimating Derivatives of a Function at a Point	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist	3.E. Provide reasons or rationales for solutions and conclusions.
2.5 Applying the Power Rule	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ A for Unit 2	
2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
2.7 Derivatives of cos x, sin x, e^x, and ln$x$	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
2.8 The Product Rule	1.E. Apply appropriate mathematical rules or procedures, with and without technology.

Unit 2: Differentiation: Definition and Basic Derivative Rules

Topic	Skill
2.9 The Quotient Rule	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ A for Unit 2	
2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.

Complete Personal Progress Checks MCQ B and FRQ B for Unit 2 Take Unit 2 Test

Unit 3: Differentiation: Composite, Implicit, and Inverse Functions	
Topic	**Skill**
3.1 The Chain Rule	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g. Use the chain rule to find the derivative of a composite function).
3.2 Implicit Differentiation	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
3.3 Differentiating Inverse Functions	3.G. Confirm that solutions are accurate and appropriate.
3.4 Differentiating Inverse Trigonometric Functions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ B for Unit 3	
3.5 Selecting Procedures for Calculating Derivatives	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g. Use the chain rule to find the derivative of a composite function).
3.6 Calculating Higher Order Derivatives	1.E. Apply appropriate mathematical rules or procedures, with and without technology.

Complete Personal Progress Checks MCQ and FRQ A for Unit 3 Take Unit 3 Test

Unit 4: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
4.1 Interpreting the Meaning of the Derivative in Context	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
4.3 Rates of Change in Applied Contexts Other Than Motion	2.A. Identify common underlying structures in problems involving different contextual situations.
4.4 Introduction to Related Rates	1.E. Apply appropriate mathematical rules or procedures, with and without technology.

4.5 Solving Related Rates Problems	3.F. Explain the meaning of mathematical solutions in context.
4.6 Approximating Values of a Function Using Local Linearity and Linearization	1.F. Explain how an approximated value relates to the actual value.
Complete Personal Progress Check FRQ A for Unit 4	
4.7 Using L'Hospital's Rule for Determining Limits and Indeterminate Forms	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Checks MCQ and FRQ B for Unit 4	
Take Unit 4 Test	

Unit 5: Differentiation: Definition and Basic Derivative Rules	
Topic	**Skill**
5.1 Using the Mean Value Theorem	3.E. Provide reasons or rationales for solutions and conclusions.
5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points	3.E. Provide reasons or rationales for solutions and conclusions.
5.3 Determining Intervals on Which a Function is Increasing or Decreasing	2.E. Describe the relationships among different representations of functions and their derivatives.
5.4 Using the First Derivative Test to Determine Relative (Local) Extrema	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ A for Unit 5	
5.5 Using the Candidates Test to Determine Absolute (Global) Extrema	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
5.6 Determining Concavity of Functions Over Their Domains	2.E. Describe the relationships among different representations of functions and their derivatives.
Complete Personal Progress Check FRQ A for Unit 5	
Unit 5: Differentiation: Definition and Basic Derivative Rules	

Topic	Skill
5.7 Using the Second Derivative Test to Determine Extrema	3.D. Apply an appropriate mathematical definition, theorem, or test.
5.8 Sketching Graphs of Functions and Their Derivatives	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Check MCQ B for Unit 5	
5.9 Connecting a Function, Its First Derivative, and Its Second Derivative	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
5.10 Introduction to Optimization Problems	2.A. Identify common underlying structures in problems involving different contextual situations.
5.11 Solving Optimization Problems	3.F. Explain the meaning of mathematical solutions in context.
5.12 Exploring Behaviors of Implicit Relations	1.E. Apply appropriate mathematical rules or procedures, with and without technology. 3.E. Provide reasons or rationales for solutions and conclusions.

Complete Personal Progress Checks MCQ C and

FRQ B for Unit 5 Take Unit 5 Test

Unit 6: Integration and Accumulation of Change

Topic	Skill
6.1 Exploring Accumulations of Change	4.B. Use appropriate units of measure.
6.2 Approximating Areas with Riemann Sums	1.F. Explain how an approximated value relates to the actual value.
6.3 Riemann Sums, Summation Notation, and Definite Integral Notation	2.C. Identify a re-expression of mathematical information presented in a given representation.
6.4 The Fundamental Theorem of Calculus and Accumulation Functions	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti- differentiation) to solve problems.
6.5 Interpreting the Behavior of Accumulation Functions Involving Area	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
Complete Personal Progress Checks MCQ A for Unit 6	

6.6 Applying Properties of Definite Integrals	3.D. Apply an appropriate mathematical definition, theorem, or test.

Unit 6: Integration and Accumulation of Change

Topic	Skill
6.7 The Fundamental Theorem of Calculus and Definite Integrals	3.D. Apply an appropriate mathematical definition, theorem, or test.
6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation	4.C. Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
6.9 Integrating Using Substitution	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ A for Unit 6	
6.10 Integrating Functions Using Long Divisions and Completing the Square	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ B for Unit 6	
6.11 Integrating Using Integration by Parts	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
6.12 Using Linear Partial Fractions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
6.13 Evaluating Improper Integrals	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
6.14 Selecting Techniques for Antidifferentiation	1.C. Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g. Use the chain rule to find the derivative of a composite function).

Complete Personal Progress Checks MCQ C and

FRQ B for Unit 6 Take Unit 6 Test

Unit 7: Differential Equations

Topic	Skill
7.1 Modeling Situations with Differential Equations	2.C. Identify a re-expression of mathematical information presented in a given representation.
7.2 Verifying Solutions for Differential Equations	3.G. Confirm that solutions are accurate and appropriate.

7.3 Sketching Slope Fields	2.C. Identify a re-expression of mathematical information presented in a given representation.
7.4 Reasoning Using Slope Fields	4.D. Use appropriate graphing techniques
7.5 Approximating Solutions Using Euler's Method	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ A for Unit 7	

Unit 7: Differential Equations	
Topic	**Skill**
7.6 Finding General Solutions Using Separation of Variables	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQs A and B for Unit 7	
7.8 Exponential Models with Differential Equations	3.G. Confirm that solutions are accurate and appropriate.
7.9 Logistic Models with Differential Equations	3.F. Explain the meaning of mathematical solutions in context.
Complete Personal Progress Check MCQ B and FRQ B for Unit 7	
Take Unit 7 Test	

Unit 8: Applications of Integration	
Topic	**Skill**
8.1 Finding the Average Value of a Function on an Interval	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
8.2 Connecting Position, Velocity, and Acceleration Using Integrals	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts	3.D. Apply an appropriate mathematical definition, theorem, or test.
8.4 Finding the Area Between Curves Expressed as Functions of x	4.C. Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).
8.5 Finding the Area Between Curves Expressed as Functions of y	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check FRQ A for Unit 8	
8.6 Finding the Area Between Curves That Intersect at More Than Two Points	2.B. Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
8.7 Volumes with Cross-Sections: Squares and Rectangles	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ A for Unit 8	

8.8 Volumes with Cross-Sections: Triangles and Semicircles	3.D. Apply an appropriate mathematical definition, theorem, or test.
8.9 Volume with Disc Method: Revolving Around the $x-$ or $y-$ axis	3.D. Apply an appropriate mathematical definition, theorem, or test.
Unit 8: Applications of Integration	
Topic	**Skill**
8.10 Volume with Disc Method: Revolving Around Other Axes	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
8.11 Volume with Washer Method: Revolving Around the $x-$ or $y-$ axis	4.E. Apply appropriate rounding procedures.
8.12 Volume with Washer Method: Revolving Around Other Axes	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
8.13 The Arc Length of a Smooth, Planar Curve and Distance Traveled	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ B and FRQ B for Unit 8	
Take Unit 8 Test	

Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions	
Topic	**Skill**
9.1 Defining and Differentiating Parametric Equations	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.
9.2 Second Derivatives of Parametric Equations	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
9.3 Finding Arc Lengths of Curves Given by Parametric Equations	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
9.4 Defining and Differentiating Vector-Valued Functions	1.D. Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, anti-differentiation) to solve problems.
9.5 Integrating Vector-Valued Functions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ A for Unit 9	
9.6 Solving Motion Problems Using Parametric and Vector-Valued Functions	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
9.7 Defining Polar Coordinates and Differentiating in Polar Form	2.D. Identify how mathematical characteristics or properties of functions are related in different representations.

Complete Personal Progress Checks FRQ A and FRQ B for Unit 9	
Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions	
Topic	**Skill**
9.8 Find the Area of a Polar Region or the Area Bounded by a Single Polar Curve	3.D. Apply an appropriate mathematical definition, theorem, or test.
9.9 Find the Area of the Region Bounded by Two Polar Curves	3.D. Apply an appropriate mathematical definition, theorem, or test.
Complete Personal Progress Check MCQ B for Unit 9	
Take Unit 9 Test	

Unit 10: Infinite Sequences and Series	
Topic	**Skill**
10.1 Defining Convergent and Divergent Infinite Series	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.2 Working with Geometric Series	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.3 The nth Term Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.4 Integral Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.5 Harmonic Series and p – Series	3.B. Identify an appropriate mathematical definition, theorem, or test to apply.
Complete Personal Progress Check MCQ A for Unit 10	
10.6 Comparison Tests for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.7 Alternating Series Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.8 Ratio Test for Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.9 Determining Absolute or Conditional Convergence	3.D. Apply an appropriate mathematical definition, theorem, or test.
10.10 Alternating Series Error Bound	1.E. Apply appropriate mathematical rules or procedures, with and without technology.
Complete Personal Progress Check MCQ B for Unit 10	
10.11 Finding Taylor Polynomial Approximations of Functions	3.D. Apply an appropriate mathematical definition, theorem, or test. 2.C. Identify a re-expression of mathematical information presented in a given representation.
10.12 Lagrange Error Bound	1.F. Explain how an approximated value relates to the actual value.
10.13 Radius and Interval of Convergence of Power Series	2.C. Identify a re-expression of mathematical information presented in a given representation.

Unit 10: Infinite Sequences and Series

Topic	Skill
10.14 Finding Taylor or Maclaurin Series for a Function	2.C. Identify a re-expression of mathematical information presented in a given representation.
Complete Personal Progress Check FRQ A for Unit 10	
10.15 Representing Functions as Power Series	3.D. Apply an appropriate mathematical definition, theorem, or test.

Complete Personal Progress Check FRQ A for Unit 10

Take Unit 10 Test

Review for AP Exam (2–3 weeks)

- Complete review of the proceeding major topics/concepts.
- Students work through problems by using published review books and College Board released MC, FRQs, and practice exams.
- Nightly practice problems from released AP Exams, including both multiple- choice and free-response questions.
- A complete practice exam (most recently released public exam) given over several class periods. This exam is graded just like the AP Exam using released scoring rubrics.

Teaching Strategies

One of the major outcomes of this course is for students to be able to work with functions represented in a variety of ways: graphically, numerically, analytically, and verbally. This is accomplished in a variety of methods, including bi-weekly homework presentations by students on the Discussion Board, downloadable problem sets with written justifications using correct mathematical nomenclature.

All exams are modeled after the AP Exam, including multiple-choice and free-response questions. All exams have both a calculator and a no calculator active section. Students are taught to round once during a problem, at the end of the problem and to three decimal places. Students are taught to develop connections/relationships between the three major themes of the course: change/limits/analysis of functions. These three themes are woven throughout the course as described in the CED. Students use their calculators to explore concepts and solidify their learning experiences. Students are exposed to a multitude of problems from various textbooks, review books, and other sources.

Activities/Projects

Students work on a number of activities/projects throughout the course to reinforce concepts and the Rule of Four. The activities below address the Four Mathematical Practices:

Mathematical Practice #1: Implementing Mathematical Processes

This activity is designed for the Accumulation of Change and Differential Equation to help students develop their own understanding of these concepts.

****The Pizza Problem Investigation**** Present this problem without initial guidance:

"A circular pizza has radius 8 inches. You cut it into sectors and eat them at a rate proportional to the remaining arc length. If you eat π inches of arc length per minute, how long until the pizza is completely eaten?"

Students must work through multiple approaches:

1. *Initial Confusion:* Most students will try basic division (16π ÷ π = 16 minutes) and get stuck when they realize the rate changes.

2. *Guided Questioning (as they struggle):*
   - "What exactly is changing as you eat the pizza?"
   - "How does the arc length relate to the central angle?"
   - "What differential equation represents this situation?"

3. *Progressive Problem-Solving:*
   - Let $\theta(t)$ = remaining central angle at time t
   - Arc length = $r\theta = 8\theta$
   - Rate equation: $\frac{d\theta}{dt} = -\frac{\pi}{8}$ (negative because decreasing)
   - Students must persist through setting up and solving this differential equation

4. *Verification Challenge:*

   Once they solve $\theta(t) = 2\pi - \left(\frac{\pi}{8}\right)t$, they must verify their answer makes physical sense.

- "What was your initial approach and why didn't it work?"
- "How did recognizing this as a differential equation help?"
- "What strategies helped you persevere when stuck?"
- Let $\theta(t) =$ remaining central angle at time t
   - Arc length = $r\theta = 8\theta$
   - Rate equation: $\frac{d\theta}{dt} = -\frac{\pi}{8}$ (negative because decreasing)
   - Students must persist through setting up and solving this differential equation. **(1.C, 1.D)**

Mathematical Practice #2: Connecting Representations

This activity is designed for connecting Integrals and Series to help students develop their own understanding of these concepts.

**** Developing the Integral Test****

Abstract Reasoning Phase:

1. *Pattern Recognition: "Let's think about Series B as $\Sigma(1/n^2)$. What function does this remind you of?"

2. *Quantitative Connection:* Graph $y = 1/x^2$ alongside the series terms as rectangles
3. *Abstract Leap:* "If we can relate the series to an integral, what might that tell us?"

Guided Development:
Students work to establish:
- When $f(x)$ is positive, decreasing, and continuous
- How $\int_1^\infty f(x)\,dx$ relates to $\Sigma f(n)$
- The logical reasoning behind why this connection works

Reasoning Tasks:
- Prove that if $\int_1^\infty f(x)\,dx$ converges, then $\Sigma f(n)$ converges
- Explain why the converse is also true
- Discuss the connection between the integral and the summation
- Apply to both harmonic and p-series examples **(2.A, 2.C)** CR4

Mathematical Practice #3: Justification

1. This activity is designed for students to justify their solution to optimization application in real- world situations.
   The Optimization Courtroom

   Argument Construction Challenge
   The Case: A rectangular garden must be enclosed with 200 feet of fencing. One side borders an existing fence (requiring no additional fencing). What dimensions maximize the area?

   Individual Argument Requirements:
   Part 1: Construct Your Argument
   Students must build a complete mathematical proof including:
   1. Variable Definition: Clearly define what each variable represents
   2. Constraint Establishment: Express the fencing constraint mathematically
   3. Objective Function: Derive the area function to optimize
   4. Critical Analysis: Find critical points with complete justification
   5. Verification: Prove the critical point yields a maximum (not minimum)

   Part 2: Critique Phase
   Present three "student solutions" with deliberate errors:
   - Solution A: Incorrect constraint equation
   - Solution B: Missing domain restrictions
   - Solution C: Assumes critical point is maximum without verification

   Individual Task: Write detailed critiques identifying mathematical errors and explaining why each argument fails. Construct counterarguments showing the correct reasoning.

Assessment Focus: Quality of mathematical argumentation, logical flow, and ability to identify flaws in reasoning. **(3.D, 3.E)** CR5

2. This activity provides a formative assessment question that gives students several functions described analytically on specified domains. The question asks students to determine whether each satisfies the hypotheses of the Mean Value Theorem, Extreme Value Theorem, and the Intermediate Value Theorem, as well as to provide written reasons for their choices. **(3.C, 3.E)** CR5

3. In another activity, the students are given information about a function and selected values of the function in a table. They are asked questions like, "Is there a value of x between a and b such that $f(x) = k$ or such that $f'(x) = m$?" The students are asked to find reasons for these conclusions. **(3.E)** CR5

4. In an instructional activity students are given a graph of f' and the graph of f''. Based on information found in these graphs, students work individually to identify local extrema for f and write justifications. Then students compare their work with a Zoom break-out session partner's, explaining their reasoning to each other **(3.E),** and both make refinements. Finally, the class works together to develop a clear and complete statement of both the first derivative test and the second derivative test based on what they have concluded from these examples. As an assessment of whether the students have learned the necessary concepts, they are given an exam question that asks them to determine whether the second derivative test applies to a variety of functions at specific x-values **(3.C).** CR5

Mathematical Practice #4: Communication and Notation

1. In this activity, the students practice notational fluency and appropriate mathematical language. Each student writes a verbal description of a rate of change (such as "The rate of change of the temperature of the pie is proportional to the difference between the temperature of the pie and the temperature of the room") and gives it to their partner, who must translate the description into a differential equation using proper notation. For that night's homework assignment, each student must submit a written explanation of how language in the verbal description suggested their differential equation. CR6 For the next night's homework assignment, they are asked to use calculus to solve the differential equation given an initial condition. The work and notation are checked. **(4.A, 4.C, 1.E)**
2. Students are given an assignment in which they are given analytical representations of the derivatives of several functions, defined over a given interval, and asked to create a candidate's table of possible values of x and $f(x)$ to find the absolute maximum value of the function over the interval. They must describe in writing the connection between the table of values and the graph of the function. (analytical, graphical, numerical) **(2.B, 2.D)** CR4

Additional Activities

1. The students are given two complicated functions expressed analytically that represent the rate of change of the populations of wolves and coyotes. They must first use the calculator to draw the graphs, then use the calculator's equation solver to find where the graphs intersect, and third use the calculator's numerical differentiation feature to estimate the slope of each graph at the intersection point. Lastly, they must set-up definite integrals to find the net increase or decrease in each population over a given time period and solve with the calculator's numerical integration feature. CR7
2. The students are asked to graph various functions and their derivatives on the calculator in order to explore the relationship between the graph of the function and the graph of its derivative and discover any useful connections (such as that the graph of f' is increasing where the graph of f is concave up). CR7

3 The students are asked to deal with a real-world economic situation confronting complex functions dealing with actual optimization issues.

Tesla Production Optimization

Scenario: Tesla's Fremont factory produces Model 3 vehicles. Production data shows:
- Production rate: $P(t) = 1000 + 200\sin(\pi t/6)$ cars per month (t in months)
- Production costs: $C'(x) = 45{,}000 - 15x + 0.002x^2$ dollars per car
- Market demand follows: $D(t) = 1200 - 50e^{(-0.1t)}$ cars per month

Use of calculator for Individual Analysis:
1. *Integration Application:* Find total cars produced in first year
2. *Optimization:* Determine monthly production level that minimizes average cost per car

Economic Decision: Recommend optimal production strategy based on mathematical analysis.

Integration Application - Total Cars Produced in First Year
Calculator Setup:
Find $\int_0^{12} P(t)\,dt = \int_0^{12} [1000 + 200\sin(\pi t/6)]\,dt$
Solution: $\int_0^{12} [1000 + 200\sin(\pi t/6)]\,dt$
$= [1000t + 200 \cdot (-6/\pi)\cos(\pi t/6)]_0^{12}$
$= [1000t - (1200/\pi)\cos(\pi t/6)]_0^{12}$
$= [1000(12) - (1200/\pi)\cos(2\pi)] - [1000(0) - (1200/\pi)\cos(0)]$
$= [12000 - (1200/\pi)(1)] - [0 - (1200/\pi)(1)]$
$= 12000 - 1200/\pi + 1200/\pi$
= **12,000 cars**
Answer: Tesla produces exactly 12,000 cars in the first year.

Optimization Application - Minimize Average Cost Per Car
Calculator Setup:
Given: $C'(x) = 45{,}000 - 15x + 0.002x^2$
First find total cost function: $C(x) = \int(45{,}000 - 15x + 0.002x^2)dx$
$C(x) = 45{,}000x - 7.5x^2 + (0.002/3)x^3 + K\ C(x)$
$= 45{,}000x - 7.5x^2 + 0.000667x^3 + K$
Average Cost Function: $AC(x) = C(x)/x = 45{,}000 - 7.5x + 0.000667x^2$
Minimize Average Cost: $AC'(x) = -7.5 + 0.001333x = 0$
Solving: $x = 7.5/0.001333$
= **5,625 cars per month**

Verification (Second Derivative Test): $AC''(x) = 0.001333 > 0$ ✓ (confirms minimum)
Minimum Average Cost: $AC(5625) = 45{,}000 - 7.5(5625) + 0.000667(5625)^2$
$= 45{,}000 - 42{,}187.5 + 21{,}093.75$
= $23,906.25 per car CR7 CR8

a. Students are introduced to using power series to estimate rational and trigonometric functions using calculator-graphing skills. Students are initially given the Taylor Series for a specific function. The power series is graphed term by term on the same viewing rectangle as the specified functions. Students visually see the power series approximation becomes a better estimate of the function as more terms are added. (2.C) CR3

b. An assignment has two polar curves. Students are asked to find the intersection points and the area closed by the two curves, $\frac{dr}{d\theta}$ or $\frac{dy}{dx}$, and interpret the meaning in the context of the problem. Students are also asked to use their graphing calculator to perform the numerical integration of $\frac{dr}{d\theta}$ or $\frac{dy}{dx}$.(3.F) CR3

c. Students will work independently and then compare their answers to identify when integration by parts is an appropriate strategy for integrating an expression and what they will choose for u and dv. They will then use integration by parts to find the integrals. We will also have a discussion covering what to do when the choices generate integrands such that we again would need to use integration by parts. Their homework will be a downloadable worksheet requiring them to select and apply the appropriate integration technique, including substitution and integration by parts. CR3

d. In this calculator-active, Zoom break-out session, students will become familiar with the graphing of trigonometric and rational functions in a real-world context. Students will find the area bound by trigonometric curves and volume of curves using cross sections to the x-axis. Students will integrate both trigonometric functions and rational functions in order to find zeros for displacement, maximums, minimums, and average values. These values will be presented with explanations to address specific questions concerning the cooling of a house, total cost of cooling a house over a particular day, number of people entering a park, dollars collected for admission to a park, and predicting when the number of people in the park is a maximum. CR8